PROOF – PRACTICE QUESTIONS

metatutor

1.
Prove algebraically that (x + 3)(2x + 2) is always an even number.

$(x+3)(2x+2) = 2(x+3)(x+1)$

therefore it is an even number.

2.
Prove algebraically that 2x(x + 5) + x(x + 2) is always a multiple of 3.

$2x^2 + 10x + x^2 + 2x$

$= 3x^2 + 12x$

$= 3(x^2 + 4)$

3.
Prove algebraically that (4x + 3)(x – 4) – 3(x + 4) is always a multiple of 4.

$4x^2 + 3x - 16x - 12 - 3x - 12$

$= 4x^2 - 16x - 24$

$= 4(x^2 - 4x - 6)$

4.
Prove algebraically that $(3x - 2)^2 + 2(6x + 1)$ is always a multiple of 3.

$9x^2 - 6x - 6x + 4 + 12x + 2$

$= 9x^2 + 6$

$= 3(3x^2 + 2)$

5.
Prove algebraically that $(x + 3)^2 - (x - 2)^2$ is always a multiple of 5.

$x^2 + 3x + 3x + 9 - (x^2 - 2x - 2x + 4)$

$= x^2 + 6x + 9 - (x^2 - 4x + 4)$

$= x^2 + 6x + 9 - x^2 + 4x - 4$

$= 10x + 5$

$= 5(2x + 1)$

6.
Prove algebraically that $(2x + 1)^2$ is always an odd number.

$$4x^2 + 2x + 2x + 1$$
$$= 4x^2 + 4x + 1$$
$$= 2(2x^2 + 2x) + 1$$

7.
Prove algebraically that $(4x - 3)^2$ is always an odd number.

$$16x^2 - 12x - 12x + 9$$
$$= 16x^2 - 24x + 8 + 1$$
$$= 2(8x^2 - 12x + 4) + 1$$

8.
Prove algebraically that $(3x - 5)^2 + x(x + 12)$ is always an odd number.

$$9x^2 - 15x - 15x + 25 + x^2 + 12x$$
$$= 10x^2 - 18x + 25$$
$$= 2(5x^2 - 9x + 12) + 1$$

9.
Prove algebraically that $(3x + 4)^2 + (x + 1)(x - 9)$ is always an odd number.

$$9x^2 + 12x + 12x + 16 + x^2 + x - 9x - 9$$
$$= 10x^2 + 16x + 7$$
$$= 2(5x^2 + 8x + 3) + 1$$

10.
Prove algebraically that $(5x + 2)^2 - (x + 1)(x - 1)$ is always an odd number.

$$25x^2 + 10x + 10x + 4 - (x^2 + x - x - 1)$$
$$= 25x^2 + 20x + 4 - (x^2 + 1)$$
$$= 25x^2 + 20x + 4 - x^2 + 1$$
$$= 24x^2 + 20x + 5$$
$$= 2(12x^2 + 10x + 2) + 1$$

11.
Prove algebraically that the sum of two consecutive integers is always odd.

$$n + n + 1 = 2n + 1$$

12.
Prove algebraically that the sum of three consecutive integers is always a multiple of 3.

$$\begin{aligned} n + n + 1 + n + 2 &= 3n + 3 \\ &= 3(n+1) \end{aligned}$$

13.
Prove algebraically that the sum of four consecutive integers is always even.

$$\begin{aligned} n + n + 1 + n + 2 + n + 3 &= 4n + 6 \\ &= 2(2n+3) \end{aligned}$$

14.
Prove algebraically that the sum of two consecutive odd integers is always a multiple of 4.

$$\begin{aligned} 2n + 1 + 2n + 3 &= 4n + 4 \\ &= 4(n+1) \end{aligned}$$

15.
Prove algebraically that the sum of three consecutive odd integers is always a multiple of 3.

$$\begin{aligned} 2n + 1 + 2n + 3 + 2n + 5 &= 6n + 9 \\ &= 3(2n+3) \end{aligned}$$

16.
Prove algebraically that the product of two consecutive odd integers is always odd.

$$\begin{aligned} (2n+1)(2n+3) &= 4n^2 + 2n + 6n + 3 \\ &= 4n^2 + 8n + 3 \\ &= 2(2n^2 + 4n + 1) + 1 \end{aligned}$$

17.
Prove algebraically that the sum of three consecutive even integers is always divisible by 6.

$$2n + 2n + 2 + 2n + 4 = 6n + 6$$
$$= 6(n+1)$$

18.
Prove algebraically that the product of two consecutive even integers is always divisible by 4.

$$2n(2n+2) = 4n^2 + 4n$$
$$= 4(n^2 + n)$$

19.
Prove algebraically that the sum of four consecutive odd integers is always divisible by 8.

$$2n + 1 + 2n + 3 + 2n + 5 + 2n + 7 = 8n + 16$$
$$= 8(n+2)$$

20.
Prove algebraically that the sum of two consecutive square numbers is always odd.

$$n^2 + (n+1)^2 = n^2 + n^2 + n + n + 1$$
$$= 2n^2 + 2n + 1$$
$$= 2(n^2 + n) + 1$$

21.
Prove algebraically that the difference of two consecutive square numbers is always odd.

$$(n+1)^2 - n^2 = n^2 + n + n + 1 - n^2$$
$$= 2n + 1$$

22.
Prove algebraically that x(x + 12) – 3(2x – 3) is always a square number.

$x^2 + 12x - 6x + 9$
$= x^2 + 6x + 9$
$= (x+3)(x+3)$
$= (x+3)^2$

23.
Prove algebraically that (2x + 9)(x + 4) – x(x + 5) is always a square number.

$2x^2 + 9x + 8x + 36 - x^2 - 5x$
$= x^2 + 12x + 36$
$= (x+6)(x+6)$
$= (x+6)^2$

24.
Prove algebraically that $(x + 8)^2 - (x + 2)^2$ is always divisible by 12.

$x^2 + 8x + 8x + 64 - (x^2 + 2x + 2x + 4)$
$= x^2 + 16x + 64 - (x^2 + 4x + 4)$
$= x^2 + 16x + 64 - x^2 - 4x - 4$
$= 12x + 60$
$= 12(x+5)$

25.
Prove algebraically that $(3x + 2)^2$ is always one more than a multiple of 3.

$9x^2 + 6x + 6x + 4 = 9x^2 + 9x + 4$
$= 9x^2 + 9x + 3 + 1$
$= 3(3x^2 + 3x + 1) + 1$

26.

Prove algebraically that $(4x + 5)^2$ is always one more than a multiple of 8.

$$16x^2 + 20x + 20x + 25 = 16x^2 + 40x + 25$$
$$= 16x^2 + 40x + 24 + 1$$
$$= 8(2x^2 + 5x + 3) + 1$$

27.

Prove algebraically that $(2x + 3)^2 - 2(8x + 7)$ is always one less than a multiple of 4.

$$4x^2 + 6x + 6x + 9 - 16x - 14$$
$$= 4x^2 - 4x - 5$$
$$= 4x^2 - 4x - 4 - 1$$
$$= 4(x^2 - x - 1) - 1$$

28.

Prove algebraically that the sum of five consecutive integers is always divisible by 5.

$$n + n + 1 + n + 2 + n + 3 + n + 4$$
$$= 5n + 10$$
$$= 5(n + 2)$$

29.
Prove algebraically that when you square an odd number and then take away the number itself you always end up with an even number.

$(2n+1)^2 - (2n+1)$

$= 4n^2 + 2n + 2n + 1 - 2n - 1$

$= 4n^2 + 2n$

$= 2(2n^2 + n)$

30.
Prove algebraically that the difference of the squares of two consecutive odd numbers is always a multiple of 8.

$(2n+3)^2 - (2n+1)^2$

$= 4n^2 + 6n + 6n + 9 - (4n^2 + 2n + 2n + 1)$

$= 4n^2 + 12n + 9 - (4n^2 + 4n + 1)$

$= 4n^2 + 12n + 9 - 4n^2 - 4n - 1$

$= 8n + 8$

$= 8(n+1)$

31.
Prove algebraically that the sum of three consecutive square numbers is always one less than a multiple of 3.

$n^2 + (n+1)^2 + (n+2)^2$

$= n^2 + n^2 + n + n + 1 + n^2 + 2n + 2n + 4$

$= 3n^2 + 6n + 5$

$= 3n^2 + 6n + 6 - 1$

$= 3(n^2 + 2n + 2) - 1$

32.

Prove algebraically that (4x + 3)(x – 1) + 7(3x + 4) is a square number for all values of x.

$4x^2+3x-4x-3+21x+28$

$=4x^2+20x+25$

$=4x^2+10x+10x+25$

$=2x(2x+5)+5(2x+5)$

$=(2x+5)(2x+5)$

$=(2x+5)^2$

33.

Prove algebraically that $(3x + 2)^2$ + x(1 – 6x) is never prime when x is positive.

$9x^2+6x+6x+4+x-6x^2$

$=3x^2+13x+4$

$=3x^2+12x+x+4$

$=3x(x+4)+1(x+4)$

$=(3x+1)(x+4)$

Because it can be Factorised, it is never prime.