ITERATION – PRACTICE QUESTIONS
CALCULATOR ALLOWED

metatutor

1.

(a) Show that a solution to the equation $x^2 + 4x - 27 = 0$ lies between 3 and 4.

$x = 3$: $3^2 + 4 \times 3 - 27 = -3$

$x = 4$: $4^2 + 4 \times 4 - 27 = 5$

0 is between -3 and 5.

(b) Show that $x^2 + 4x - 27 = 0$ can be rearranged into the equation $x = \sqrt{27 - 4x}$.

$x^2 = 27 - 4x$

$x = \sqrt{27 - 4x}$

(c) Using the iterative formula $x_{n+1} = \sqrt{27 - 4x_n}$ with $x_0 = 3$, find x_1, x_2, x_3 and x_4.

$x_0 = 3$

$x_1 = \sqrt{27 - 4 \times 3} = \sqrt{15}$

$x_2 = \sqrt{27 - 4x_1} = 3.392354141$

$x_3 = 3.664776042$

$x_4 = 3.51296112$

2.

(a) Show that a solution to the equation $x^2 - 3x - 9 = 0$ lies between 4 and 5.

$x = 4$: $4^2 - 3 \times 4 - 9 = -5$

$x = 5$: $5^2 - 3 \times 5 - 9 = 1$

0 is between -5 and 1.

(b) Show that $x^2 - 3x - 9 = 0$ can be rearranged into the equation $x = \sqrt{3x + 9}$.

$x^2 = 3x + 9$

$x = \sqrt{3x + 9}$

(c) Using the iterative formula $x_{n+1} = \sqrt{3x_n + 9}$ with $x_0 = 2$, find x_1, x_2, x_3 and x_4.

$x_0 = 2$

$x_1 = \sqrt{3 \times 2 + 9} = \sqrt{15}$

$x_2 = 4.540809403$

$x_3 = 4.756304049$

$x_4 = 4.823786608$

3.

(a) Show that a solution to the equation $x^3 - 2x = 9$ lies between 2 and 3.

$x=2: 2^3 - 2\times2 = 4$

$x=3: 3^3 - 2\times3 = 21$

9 is between 4 and 21

(b) Show that $x^3 - 2x = 9$ can be rearranged into the equation $x = \sqrt[3]{2x+9}$.

$x^3 = 2x + 9$

$x = \sqrt[3]{2x+9}$

(c) Using the iterative formula $x_{n+1} = \sqrt[3]{2x_n + 9}$ with $x_0 = 2$, use 3 iterations to find an estimate for the solution of $x^3 - 2x = 9$.

$x_0 = 2$

$x_1 = \sqrt[3]{2\times2+9} = 2.351334688$

$x_2 = 2.392957909$

$x_3 = 2.397794027$

2.40

4.

(a) Show that a solution to the equation $x^3 + 10x = 8$ lies between 0 and 1.

$x=0: 0^3 + 10\times0 = 0$

$x=1: 1^3 + 10\times1 = 11$

8 is between 0 and 11.

(b) Show that $x^3 + 10x = 8$ can be rearranged into the equation $x = \frac{8-x^3}{10}$.

$10x = 8 - x^3$

$x = \frac{8-x^3}{10}$

(c) Using the iterative formula $x_{n+1} = \frac{8-x_n^3}{10}$ with $x_0 = 0$ use 4 iterations to find an estimate for the solution of $x^3 + 10x = 8$.

$x_0 = 0$

$x_1 = \frac{8-0^3}{10} = 0.8$

$x_2 = 0.7488$

$x_3 = 0.7580146762$

$x_4 = 0.7564455519$

0.76

5.

(a) Show that a solution to the equation $7x - x^2 - 8 = 0$ lies between 1 and 2.

$x=1: \ 7\times1-1^2-8=-2$

$x=2: \ 7\times2-2^2-8=2$

0 is between -2 and 2.

(b) Show that $7x - x^2 - 8 = 0$ can be rearranged into the equation $x = \frac{x^2+8}{7}$.

$7x = x^2+8$

$x = \frac{x^2+8}{7}$

(c) Using the iterative formula $x_{n+1} = \frac{x_n^2+8}{7}$ and a suitable value of x_0, find the solution of the equation $7x - x^2 - 8 = 0$ to 3 decimal places.

$x_0 = 1$ (or you could choose 2)

$x_1 = 1.285714$

$x_2 = 1.3790008746$

$\vdots$

1.438

6.

(a) Show that a solution to the equation $12x - 3x^2 = 10$ lies between 1 and 2.

$x=1: \ 12\times1-3\times1^2=9$

$x=2: \ 12\times2-3\times2^2=12$

10 is between 9 and 12.

(b) Show that $12x - 3x^2 = 10$ can be rearranged into the equation $x = \frac{3x^2+10}{12}$.

$12x = 3x^2+10$

$x = \frac{3x^2+10}{12}$

(c) Using the iterative formula $x_{n+1} = \frac{3x_n^2+10}{12}$ and a suitable value of x_0, find the solution of the equation $12x - 3x^2 = 10$ to 2 decimal places.

$x_0 = 1$ (or 2)

$x_1 = 1.083$

$x_2 = 1.1267361$

$\vdots$

1.18

7.

(a) Show that a solution to the equation $2x^2 + 7x - 31 = 0$ lies between 2 and 3.

$x=2: 2\times2^2+7\times2-31=-9$

$x=3: 2\times3^2+7\times3-31=8$

0 is between −9 and 8

(b) Show that $2x^2 + 7x - 31 = 0$ can be rearranged into the equation $x = \sqrt{\frac{31-7x}{2}}$.

$2x^2 = 31-7x$

$x^2 = \frac{31-7x}{2}$ $\quad x = \sqrt{\frac{31-7x}{2}}$

(c) Using the iterative formula $x_{n+1} = \sqrt{\frac{31-7x_n}{2}}$ and a suitable value of x_0, find the solution of the equation $2x^2 + 7x - 31 = 0$ to 2 decimal places.

$x_0 = 2$ (or 3)

$x_1 = 2.915475947$

$x_2 = 2.301267951$

⋮

2.56

8.

(a) Show that a solution to the equation $3x^2 + 12x - 51 = 0$ lies between 2 and 3.

$x=2: 3\times2^2+12\times2-51=-15$

$x=3: 3\times3^2+12\times3-51=12$

0 is between −15 and 12

(b) Show that $3x^2 + 12x - 51 = 0$ can be rearranged into the equation $x = \sqrt{17-4x}$.

$3x^2 = 51-12x$ $\quad x^2 = 17-4x$

$x^2 = \frac{51-12x}{3}$ $\quad x = \sqrt{17-4x}$

(c) Using the iterative formula $x_{n+1} = \sqrt{17-4x_n}$ and a suitable value of x_0, find the solution of the equation $3x^2 + 12x - 51 = 0$ to 3 significant figures.

$x_0 = 2$

$x_1 = 3$

$x_2 = 2.236067977$

$x_3 = 2.838261456$

⋮

2.58

9.

(a) Show that a solution to the equation $4x^3 - 15x = 75$ lies between 3.1 and 3.2.

$x = 3.1$; $4 \times 3.1^3 - 15 \times 3.1 = 72.664$ 75 is between 72 and 83

$x = 3.2$; $4 \times 3.2^3 - 15 \times 3.2 = 83.072$

(b) Show that $4x^3 - 15x = 75$ can be rearranged into the equation $x = \sqrt[3]{\frac{15x+75}{4}}$.

$4x^3 = 15x + 75$

$x^3 = \frac{15x+75}{4}$ $\quad x = \sqrt[3]{\frac{15x+75}{4}}$

(c) Using the iterative formula $x_{n+1} = \sqrt[3]{\frac{15x_n+75}{4}}$ and a suitable value of x_0, find the solution of the equation $4x^3 - 15x = 75$ to 4 significant figures.

$x_0 = 3.1$ (or 3.2)

$x_1 = 3.120125735$

$x_2 = 3.122707746$

$x_3 = 3.123038694$

⋮

3.123

10.

(a) Show that a solution to the equation $x^3 + 4x = 4$ lies between 0 and 1.

$x = 0$; $0^3 + 4 \times 0 = 0$ 4 is between 0 and 5

$x = 1$; $1^3 + 4 \times 1 = 5$

(b) Show that $x^3 + 4x = 4$ can be rearranged into the equation $x = 1 - \frac{x^3}{4}$.

$4x = 4 - x^3$

$x = \frac{4 - x^3}{4}$ $\quad x = 1 - \frac{x^3}{4}$

(c) Using a suitable iterative formula and a suitable x_0, find the solution to 4 significant figures.

$x_0 = 0$ (or $x_0 = 1$)

$x_1 = 1$

$x_2 = 0.75$

$x_3 = 0.89453125$

⋮

0.8477

11.

(a) Show that a solution to the equation $x^3 + 9x^2 = 60$ lies between -9 and -8.

$x = -9$: $(-9)^3 + 9(-9)^2 = 0$

$x = -8$: $(-8)^3 + 9(-8)^2 = 64$

60 is between 0 and 64

(b) Show that $x^3 + 9x^2 = 60$ can be rearranged into the equation $x = \frac{60}{x^2} - 9$.

$x^3 = 60 - 9x^2$

$x = \frac{60 - 9x^2}{x^2}$ $\quad x = \frac{60}{x^2} - 9$

(c) Using a suitable iterative formula and a suitable x_0, find the solution to 4 significant figures.

$x_0 = -9$ (or -8)

$x_1 = -8.259$

$x_2 = -8.120432745$

$x_3 = -8.090101597$

$\vdots$

-8.081

12.

(a) Show that a solution to the equation $x^3 + 6x - 11 = 0$ lies between 1 and 2.

$x = 1$: $1^3 + 6 \times 1 - 11 = -4$

$x = 2$: $2^3 + 6 \times 2 - 11 = 9$

0 is between -4 and 9

(b) Show that $x^3 + 6x - 11 = 0$ can be rearranged into the equation $x = \frac{11}{x^2+6}$.

$x^3 + 6x = 11$

$x(x^2 + 6) = 11$ $\quad x = \frac{11}{x^2+6}$

(c) Using a suitable iterative formula and a suitable x_0, find the solution to 4 significant figures.

$x_0 = 1$ (or -2)

$x_1 = 1.571428$

$x_2 = 1.298795180722 6\ldots$

$x_3 = 1.431011783$

$\vdots$

1.388