

FUNCTIONS - PRACTICE QUESTIONS

1.

f and g are functions.

$$f(x) = 2x + 5$$

$$g(x) = 6x^2$$

(a) Find $f(3)$

$$2 \times 3 + 5 = \underline{11}$$

(b) Find $g(-1)$

$$6 \times (-1)^2 = \underline{6}$$

(c) Find $f(5) - g(1)$

$$f(5) = 2 \times 5 + 5 = 15$$

$$15 - 6 = \underline{9}$$

$$g(1) = 6 \times 1^2 = 6$$

(d) Find $\frac{g(3)}{f(2)}$

$$g(3) = 6 \times 3^2 = 54$$

$$54 \div 9 = \underline{6}$$

$$f(2) = 2 \times 2 + 5 = 9$$

2.

a and b are functions.

$$a(x) = 2x - 4$$

$$b(x) = 4x$$

(a) Find $a(-1)$

$$2 \times -1 - 4 = \underline{-6}$$

(b) Find $b(2) \times a(1)$

$$b(2) = 4 \times 2 = 8$$

$$8 \times -2 = \underline{-16}$$

$$a(1) = 2 \times 1 - 4 = -2$$

(c) Find $bb(1)$

$$b(1) = 4 \times 1 = 4$$

$$b(4) = 4 \times 4 = \underline{16}$$

(d) Find $ab(3)$

$$b(3) = 4 \times 3 = 12$$

$$a(12) = 2 \times 12 - 4 = \underline{20}$$

3.

c and d are functions.

$$c(x) = \frac{x}{2}$$

$$d(x) = x^2 + 1$$

(a) Find $c(4) + d(4)$

$$c(4) = 4 \div 2 = 2$$

$$d(4) = 4^2 + 1 = 17$$

$$2 + 17 = \underline{19}$$

(b) Find $cd(3)$

$$d(3) = 3^2 + 1 = 10$$

$$c(10) = \frac{10}{2} = \underline{5}$$

(c) Solve $d(x) = 26$

$$x^2 + 1 = 26$$

$$x^2 = 25$$

$$\underline{x = \pm 5}$$

4.

e and f are functions.

$$e(x) = 3x$$

$$f(x) = x + 1$$

(a) Find $ef(x)$

$$ef(x) = e(f(x)) = e(x+1) = 3(x+1) = \underline{3x+3}$$

(b) Find $fe(x)$

$$fe(x) = f(e(x)) = f(3x) = \underline{3x+1}$$

(c) Find $ef(2) + fe(5)$

$$ef(2) = 3 \times 2 + 3 = 9$$

$$fe(5) = 3 \times 5 + 1 = 16$$

$$9 + 16 = \underline{25}$$

5.

g and h are functions.

$$g(x) = 2x - 3$$

$$h(x) = x + 5$$

(a) Find $gh(x)$

$$\begin{aligned} gh(x) &= g(h(x)) = g(x+5) = 2(x+5) - 3 \\ &= 2x + 10 - 3 \\ &= \underline{2x + 7} \end{aligned}$$

(b) Find $hg(x)$

$$\begin{aligned} hg(x) &= h(g(x)) = h(2x-3) = 2x-3+5 \\ &= \underline{2x+2} \end{aligned}$$

(c) Find $hh(x)$

$$\begin{aligned} hh(x) &= h(h(x)) = h(x+5) \\ &= x+5+5 \\ &= \underline{x+10} \end{aligned}$$

6.

i and j are functions.

$$i(x) = 3x + 10$$

$$j(x) = 2x - 5$$

(a) Find $ij(x)$

$$\begin{aligned} ij(x) &= i(j(x)) = i(2x-5) = 3(2x-5) + 10 \\ &= 6x - 15 + 10 \\ &= \underline{6x - 5} \end{aligned}$$

(b) Find $ji(x)$

$$\begin{aligned} ji(x) &= j(i(x)) = j(3x+10) = 2(3x+10) - 5 \\ &= 6x + 20 - 5 \\ &= \underline{6x + 15} \end{aligned}$$

(c) Solve $ji(x) = 45$.

$$\begin{aligned} 6x + 15 &= 45 \\ 6x &= 30 \\ \underline{x} &= \underline{5} \end{aligned}$$

7.

k and l are functions.

$$k(x) = 4x$$

$$l(x) = \frac{3x-1}{2}$$

(a) Find $lk(x)$ $lk(x) = l(k(x)) = \frac{3(4x)-1}{2} = \frac{12x-1}{2}$

(b) Find $kl(x)$ $kl(x) = k(l(x)) = 4\left(\frac{3x-1}{2}\right) = 2(3x-1)$
 $= \underline{6x-1}$

(c) Solve $kk(x) = 64$.

$$kk(x) = k(k(x)) = 4(4x) = 16x$$

$$16x = 64$$

$$\underline{x = 4}$$

8.

m, n and o are functions.

$$m(x) = 5x + 1$$

$$n(x) = \frac{x}{3}$$

$$o(x) = 2x - 9$$

(a) Find $m^{-1}(x)$

$$y = 5x + 1$$
$$y - 1 = 5x$$
$$\frac{y-1}{5} = x$$

$$m^{-1}(x) = \frac{x-1}{5}$$

(b) Find $n^{-1}(x)$

$$y = \frac{x}{3}$$
$$3y = x$$

$$n^{-1}(x) = 3x$$

(c) Find $o^{-1}(x)$

$$y = 2x - 9$$
$$y + 9 = 2x$$
$$\frac{y+9}{2} = x$$

$$o^{-1}(x) = \frac{x+9}{2}$$

9.

p, q and r are functions.

$$p(x) = 6x - 1$$

$$q(x) = 3x$$

$$r(x) = x^2 + 10$$

(a) Find $p^{-1}(x)$

$$\begin{aligned}y &= 6x - 1 \\y + 1 &= 6x \\ \frac{y+1}{6} &= x\end{aligned}$$

$$p^{-1}(x) = \frac{x+1}{6}$$

(b) Find $q^{-1}(x)$

$$\begin{aligned}y &= 3x \\ \frac{y}{3} &= x\end{aligned}$$

$$q^{-1}(x) = \frac{x}{3}$$

(c) Find $r^{-1}(x)$

$$\begin{aligned}y &= x^2 + 10 \\y - 10 &= x^2 \\ \sqrt{y-10} &= x\end{aligned}$$

$$r^{-1}(x) = \sqrt{x-10}$$

10.

t, u and v are functions.

$$t(x) = \frac{x+3}{2}$$

$$u(x) = 3x^3$$

$$v(x) = \frac{3x-2}{5}$$

(a) Find $t^{-1}(x)$

$$\begin{aligned}y &= \frac{x+3}{2} \\ 2y &= x+3 \\ 2y-3 &= x\end{aligned}$$

$$t^{-1}(x) = 2x - 3$$

(b) Find $u^{-1}(x)$

$$\begin{aligned}y &= 3x^3 \\ \frac{y}{3} &= x^3 \\ \sqrt[3]{\frac{y}{3}} &= x\end{aligned}$$

$$u^{-1}(x) = \sqrt[3]{\frac{x}{3}}$$

(c) Find $v^{-1}(x)$

$$\begin{aligned}y &= \frac{3x-2}{5} \\ 5y &= 3x-2 \\ 5y+2 &= 3x \\ \frac{5y+2}{3} &= x\end{aligned}$$

$$v^{-1}(x) = \frac{5x+2}{3}$$

11.

a, b and c are functions.

$$a(x) = 4x + 1$$

$$b(x) = 2x^2 - 1$$

$$c(x) = \frac{5x}{4}$$

(a) Find $c(12)$

$$\frac{5 \times 12}{4} = \frac{60}{4} = \underline{15}$$

(b) Solve $a(x) = 37$

$$4x + 1 = 37$$

$$4x = 36$$

$$\underline{x = 9}$$

(c) Find $ac(x)$

$$\begin{aligned} ac(x) &= a(c(x)) = a\left(\frac{5x}{4}\right) = 4\left(\frac{5x}{4}\right) + 1 \\ &= \underline{5x + 1} \end{aligned}$$

(d) Find $b^{-1}(x)$

$$\begin{aligned} y &= 2x^2 - 1 \\ y + 1 &= 2x^2 \\ \sqrt{\frac{y+1}{2}} &= x \end{aligned}$$

$$b^{-1}(x) = \sqrt{\frac{x+1}{2}}$$

12.

d, e and f are functions.

$$d(x) = \frac{x^2 + 5}{3}$$

$$e(x) = \sqrt{x+1}$$

$$f(x) = 3x - 10$$

(a) Find $fe(35)$

$$e(35) = \sqrt{35+1} = \sqrt{36} = 6$$

$$f(6) = 3 \times 6 - 10 = \underline{8}$$

(b) Find $d^{-1}(x)$

$$3y = x^2 + 5$$

$$3y - 5 = x^2$$

$$\sqrt{3y-5} = x$$

$$d^{-1}(x) = \sqrt{3x-5}$$

(c) Find $e^{-1}(x)$

$$y = \sqrt{x+1}$$

$$y^2 = x + 1$$

$$y^2 - 1 = x$$

$$e^{-1}(x) = x^2 - 1$$

13.

g, h and i are functions.

$$g(x) = 3x + 4$$

$$h(x) = x^2 - 10$$

$$i(x) = 2x^2 + 5x - 4$$

(a) Find $hg(x)$. Express your answer in the form $ax^2 + bx + c$.

$$\begin{aligned} hg(x) &= h(g(x)) = h(3x+4) = (3x+4)^2 - 10 \\ &= 9x^2 + 12x + 12x + 16 - 10 \\ &= \underline{9x^2 + 24x + 6} \end{aligned}$$

(b) Solve $i(x) = h(x)$

$$\begin{aligned} 2x^2 + 5x - 4 &= x^2 - 10 \\ x^2 + 5x + 6 &= 0 \\ (x+2)(x+3) &= 0 \end{aligned}$$

$$\underline{x = -2 \text{ or } -3}$$

14.

j and k are functions.

$$j(x) = \sqrt{x} + 4$$

$$k(x) = x^2 + 6x + 9$$

(a) Show that $k(x+2) - k(x) = 4x + 16$.

$$\begin{aligned} k(x+2) &= (x+2)^2 + 6(x+2) + 9 \\ &= x^2 + 4x + 4 + 6x + 12 + 9 \\ &= x^2 + 10x + 25 \end{aligned}$$

$$\begin{aligned} x^2 + 10x + 25 - (x^2 + 6x + 9) &= x^2 + 10x + 25 - x^2 - 6x - 9 \\ &= \underline{4x + 16} \end{aligned}$$

(b) Solve $jk(x) = 10$

$$j(k(x)) = j(x^2 + 6x + 9) = \sqrt{x^2 + 6x + 9} + 4$$

$$\sqrt{x^2 + 6x + 9} + 4 = 10$$

$$\sqrt{x^2 + 6x + 9} = 6$$

$$x^2 + 6x + 9 = 36$$

$$x^2 + 6x - 27 = 0$$

$$(x+9)(x-3) = 0$$

$$\underline{x = -9 \text{ or } 3}$$

15.

m, n and o are functions.

$$m(x) = \sqrt{\frac{x}{2}}$$

$$n(x) = 5x^2 - 48$$

$$o(x) = 18x^6$$

(a) Solve $m^{-1}(x) = n(x)$

$$y = \sqrt{\frac{x}{2}}$$

$$y^2 = \frac{x}{2}$$

$$2y^2 = x$$

$$m^{-1}(x) = 2x^2$$

$$2x^2 = 5x^2 - 48$$

$$48 = 3x^2$$

$$16 = x^2$$

$$x = \pm 4$$

(b) Show that $mo(x) = 3x^3$

$$m \circ (x) = m(o(x)) = \sqrt{\frac{18x^6}{2}} = \sqrt{9x^6} = \underline{3x^3}$$

16.

p, q and r are functions.

$$p(x) = \frac{x}{x+1}$$

$$q(x) = x^2 + 2x - 3$$

(a) Find $p^{-1}(x)$

$$y = \frac{x}{x+1}$$

$$y(x+1) = x$$

$$yx + y = x$$

$$y = x - yx$$

$$y = x(1-y)$$

$$\frac{y}{1-y} = x$$

$$p^{-1}(x) = \frac{x}{1-x}$$

(b) Solve $q(x+2) = 0$

$$q(x+2) = (x+2)^2 + 2(x+2) - 3$$

$$= x^2 + 4x + 4 + 2x + 4 - 3$$

$$= x^2 + 6x + 5$$

$$x^2 + 6x + 5 = 0$$

$$(x+1)(x+5) = 0$$

$$\underline{x = -1 \text{ or } -5}$$

17.

f and g are functions.

$$f(x) = ax^2 + b$$

$$g(x) = bx$$

Given that $f(2) = 14$ and $g(4) = 8$, show that $gf(1) = 10$.

$$g(4) = 8$$

$$4b = 8$$

$$b = 2$$

$$f(x) = ax^2 + 2$$

$$f(2) = 4a + 2 = 14$$

$$4a = 12$$

$$a = 3$$

$$f(x) = 3x^2 + 2$$

$$g(x) = 2x$$

$$f(1) = 3 \times 1^2 + 2$$

$$= 5$$

$$g(5) = 2 \times 5 = \underline{10}$$

18.

h and i are functions.

$$h(x) = \sqrt{\frac{3x+1}{4}}$$

$$i(x) = 4x^2$$

(a) Find $h^{-1}(x)$

$$y = \sqrt{\frac{3x+1}{4}}$$

$$4y^2 = 3x+1$$

$$4y^2 - 1 = 3x$$

$$x = \frac{4y^2 - 1}{3}$$

$$h^{-1}(x) = \frac{4x^2 - 1}{3}$$

(b) Solve $ih(x) = 19$

$$ih(x) = i(h(x)) = i\left(\sqrt{\frac{3x+1}{4}}\right) = 4\left(\sqrt{\frac{3x+1}{4}}\right)^2$$

$$= 4\left(\frac{3x+1}{4}\right)$$

$$= 3x+1$$

$$3x+1 = 19$$

$$3x = 18$$

$$x = \underline{6}$$

19.

j and k are functions.

$$j(x) = x^2 + 1$$

$$k(x) = 2 + x$$

Solve $jk(x) = kj(x)$.

$$\begin{aligned} jk(x) &= j(k(x)) = j(2+x) = (2+x)^2 + 1 \\ &= 4 + 4x + x^2 + 1 \\ &= x^2 + 4x + 5 \end{aligned}$$

$$\begin{aligned} kj(x) &= k(j(x)) = k(x^2+1) = 2 + x^2 + 1 \\ &= x^2 + 3 \end{aligned}$$

$$x^2 + 4x + 5 = x^2 + 3$$

$$4x = -2$$

$$x = -\frac{1}{2}$$

20.

m and n are functions.

$$m(x) = \sqrt{ax+b}$$

$$n(x) = \frac{ax+1}{3}$$

Given that $m^{-1}(4) = 3$ and $n^{-1}(3) = 2$, find $mn(2)$.

(Find $n^{-1}(x)$)

$$y = \frac{ax+1}{3}$$

$$3y = ax+1$$

$$3y-1 = ax$$

$$\frac{3y-1}{a} = x$$

$$n^{-1}(x) = \frac{3x-1}{a}$$

$$n^{-1}(3) = \frac{3 \times 3 - 1}{a}$$

$$2 = \frac{8}{a}$$

$$a = 4$$

$$m(x) = \sqrt{4x+b}$$

$$y = \sqrt{4x+b}$$

$$y^2 = 4x+b$$

$$\frac{y^2-b}{4} = x$$

$$m^{-1}(x) = \frac{x^2-b}{4}$$

$$m^{-1}(4) = \frac{4^2-b}{4}$$

$$3 = \frac{16-b}{4}$$

$$12 = 16-b$$

$$b = 4$$

$$m(x) = \sqrt{4x+4}$$

$$n(x) = \frac{4x+1}{3}$$

$$n(2) = \frac{4 \times 2 + 1}{3} = 3$$

$$m(3) = \sqrt{4 \times 3 + 4}$$

$$= \sqrt{16}$$

$$= \underline{\underline{\pm 4}}$$