DIRECT AND INVERSE PROPORTION – PRACTICE QUESTIONS

1. 
B is directly proportional to A.
When A = 6, B = 72.

(a) Find an equation for B in terms of A.

\[ B = kA \]
\[ 72 = k \times 6 \]
\[ 12 = k \]

\[ D = 12A \]

(b) Find the value of B when A = 4.5.

\[ B = 4.5 \times 12 = 54 \]

2. 
D is directly proportional to C.
When C = 5, D = 90.

(a) Find an equation for D in terms of C.

\[ D = kC \]
\[ 90 = k \times 5 \]
\[ 18 = k \]

\[ D = 18C \]

(b) Find the value of D when C = 7.

\[ D = 18 \times 7 = 126 \]

(c) Find the value of C when D = 126.

\[ 126 = 18C \]
\[ C = 7 \]
3.  
F is directly proportional to $E^2$.  
When $E = 3$, $F = 108$.  
(a) Find an equation for $F$ in terms of $E$.  
\[ F = kE^2 \]
\[ 108 = k \times 3^2 \]
\[ 108 = 9k \]
\[ 12 = k \]
\[ F = 12E^2 \]
(b) Find the value of $F$ when $E = 2$.  
\[ F = 12 \times 2^2 = 48 \]
(c) Find the value of $E$ when $F = 1200$.  
\[ 1200 = 12E^2 \]
\[ 100 = E^2 \]
\[ E = 10 \]
4.  
G is directly proportional to $\sqrt{H}$.  
When $H = 400$, $G = 60$.  
(a) Find an equation for $G$ in terms of $H$.  
\[ G = k\sqrt{H} \]
\[ 60 = k \times \sqrt{400} \]
\[ 60 = 20k \]
\[ k = 3 \]
\[ G = 3\sqrt{H} \]
(b) Find the value of $G$ when $H = 64$.  
\[ G = 3 \times \sqrt{64} = 24 \]
(c) Find the value of $H$ when $G = 75$.  
\[ 75 = 3\sqrt{H} \]
\[ 25 = \sqrt{H} \]
\[ 625 = H \]
5. 
Q is inversely proportional to P.
When \( P = 0.5 \), \( Q = 16 \).

(a) Find an equation for \( Q \) in terms of \( P \).
\[
Q = \frac{k}{P} \\
16 = \frac{k}{0.5} \\
8 = k
\]

(b) Find the value of \( Q \) when \( P = 4 \).
\[
Q = \frac{8}{4} = 2
\]

(c) Find the value of \( P \) when \( Q = 1.6 \).
\[
1.6 = \frac{8}{P} \\
1.6P = 8 \\
P = \frac{8}{1.6} = 5
\]

6. 
M is inversely proportional to \( N \).
When \( N = 6 \), \( M = 11 \).

(a) Find an equation for \( M \) in terms of \( N \).
\[
M = \frac{k}{N} \\
11 = \frac{k}{6} \\
66 = k \\
M = \frac{66}{N}
\]

(b) Find the value of \( N \) when \( M = 132 \).
\[
132 = \frac{66}{N} \\
N = \frac{66}{132} = \frac{1}{2}
\]

(c) Find the value of \( M \) when \( N = 22 \).
\[
M = \frac{66}{22} = 3
\]
7. O is inversely proportional to $P^3$. When $P = 3$, $O = 2$.

(a) Find an equation for $O$ in terms of $P$.

$$O = \frac{k}{P^3}$$

$$2 = \frac{k}{3^3}$$
$$2 = \frac{k}{27}$$
$$54 = k$$

(b) Find the value of $O$ when $P = 2$.

$$0 = \frac{54}{2^3} = 6.75$$

(c) Find the value of $P$ when $O = 432$.

$$432 = \frac{54}{P^3}$$
$$P^3 = \frac{54}{432}$$
$$P = \sqrt[3]{\frac{54}{432}} = \frac{1}{2}$$

8. T is inversely proportional to $\sqrt{U}$. When $U = 16$, $T = 20$.

(a) Find an equation for $T$ in terms of $U$.

$$T = \frac{k}{\sqrt{U}}$$

$$20 = \frac{k}{\sqrt{16}}$$
$$80 = k$$

(b) Find the value of $U$ when $T = 160$.

$$160 = \frac{80}{\sqrt{U}}$$
$$\sqrt{U} = \frac{80}{160}$$
$$U = \left(\frac{80}{160}\right)^2 = \left(\frac{1}{4}\right)$$

(c) Find the value of $T$ when $U = 64$.

$$T = \frac{80}{\sqrt{64}} = \frac{80}{8} = 10$$
9.
W is directly proportional to $V^2$.
When $V = 5, W = 400$.

(a) Find an equation for $W$ in terms of $V$.

\[
\begin{align*}
W &= kV^2 \\
400 &= k \times 5^2 \\
400 &= 25k \\
k &= 16
\end{align*}
\]

\[
W = 16V^2
\]

(b) Find the value of $W$ when $V = 1.5$.

\[
W = 16 \times 1.5^2 = 36
\]

(c) Find the value of $V$ when $W = 6$.

\[
6 = 16V^2 \quad \frac{6}{16} = V^2 \quad V = \sqrt{\frac{6}{16}} = 0.612372 \ldots
\]

10.
$Y$ is inversely proportional to $\sqrt[3]{X}$.
When $X = 125, Y = 22$.

(a) Find an equation for $Y$ in terms of $X$.

\[
Y = \frac{k}{\sqrt[3]{X}}
\]

\[
22 = \frac{k}{\sqrt[3]{125}}
\]

\[
k = 22 \times \sqrt[3]{125}
\]

\[
k = 110
\]

\[
Y = \frac{110}{\sqrt[3]{X}}
\]

(b) Find the value of $Y$ when $X = 1,000$.

\[
Y = \frac{110}{\sqrt[3]{1000}} = \frac{11}{1}
\]

(c) Find the value of $X$ when $Y = 13$. Give your answer to 3 significant figures.

\[
13 = \frac{110}{\sqrt[3]{X}} \quad 3\sqrt[3]{X} = \frac{110}{13} \quad X = \left(\frac{110}{13}\right)^3 = 605.826 \ldots
\]

\[
X = 606
\]
11.
\( e \propto r^2 \).
Complete the table.

<table>
<thead>
<tr>
<th>( e )</th>
<th>750</th>
<th>120</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>10</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

\[
e = Kr^2
\]

\[
750 = K \times 10^2
\]

\[
\frac{750}{100} = K
\]

\[
k = 7.5
\]

\[
e = 7.5r^2
\]

\[
r = 4, \ e = 7.5 \times 4^2 = 120
\]

\[
e = 30, \ 30 = 7.5 \times r^2
\]

\[
4 = r^2
\]

\[
r = 2
\]

12.
\( L \propto \frac{1}{M} \).
Complete the table.

<table>
<thead>
<tr>
<th>( L )</th>
<th>0.5</th>
<th>12</th>
<th>0.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )</td>
<td>6</td>
<td>0.25</td>
<td>20</td>
</tr>
</tbody>
</table>

\[
L = \frac{k}{M}
\]

\[
0.15 = \frac{k}{20}
\]

\[
3 = k
\]

\[
L = \frac{3}{M}
\]

\[
L = 0.5, \ 0.5 = \frac{3}{M}
\]

\[
M = \frac{3}{0.5}
\]

\[
= 6
\]

\[
M = 0.25, \ L = \frac{3}{0.25}
\]

\[
= 12
\]
13.
\( P \propto \frac{1}{t^2} \).

Complete the table.

<table>
<thead>
<tr>
<th>( P )</th>
<th>0.0125</th>
<th>0.8</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>4</td>
<td>0.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\[
P = \frac{k}{t^2}
\]

\[
20 = \frac{k}{0.1^2}
\]

\[
0.2 = k
\]

\[
P = \frac{0.2}{t^2}
\]

\[
t = 4, \quad P = \frac{0.2}{4^2} = 0.0125
\]

\[
\rho = 0.8, \quad 0.8 = \frac{0.2}{t^2} \]

\[
t^2 = \frac{0.2}{0.8}
\]

\[
t = \frac{1}{2} = 0.5
\]

14.
The weight (in grams) of a piece of wire is directly proportional to its length (in centimetres).

A piece of wire weighs 100 grams and is 25 centimetres long.

Find the weight of a piece of wire which is 60 centimetres long.

\[
W = kL
\]

\[
100 = k \times 25
\]

\[
k = 4
\]

\[
W = 4L
\]

\[
L = 60, \quad W = 4 \times 60 = 240 \text{ grams}
\]

15.
The force \( F \) (in Newtons) between two magnets is inversely proportional to the distance \( D \) (in metres) between them.

When the magnets are 0.8 m apart, the force between them is 150 Newtons.

Find the distance between the two magnets when the force between them is 220 Newtons.

\[
F = \frac{k}{D}
\]

\[
150 = \frac{k}{0.8}
\]

\[
k = 120
\]

\[
F = \frac{120}{D}
\]

\[
F = 220,
\]

\[
220 = \frac{120}{D}
\]

\[
D = \frac{120}{220} = 0.545454\ldots \text{ m}
\]

\[
D = \frac{120}{220} = 55 \text{ cm}
\]
16. The speed that a long distance runner runs at is inversely proportional to the time they have been running for. After running for 2 hours, the runner is running at 3 metres per second. Work out the speed at which the runner is running after 150 minutes.

\[
S = \frac{k}{T}
\]
\[
3 = \frac{k}{2}
\]
\[
k = 6
\]
\[
S = \frac{6}{2.5} = 2.4 \text{ m/s}
\]

17. The distance, D (in kilometres), travelled by a space shuttle is directly proportional to the square of the amount of fuel carried, F (in gallons). On Mission 1, the shuttle carried 500 gallons of fuel and travelled \(6.5 \times 10^5\) kilometres. On Mission 2, the shuttle travelled \(8.8 \times 10^6\) kilometres. Find the number of gallons of fuel carried by the shuttle on Mission 2, to the nearest gallon.

\[
D = kF^2
\]
\[
6.5 \times 10^5 = k \times 500^2
\]
\[
k = 2.6
\]
\[
D = 2.6F^2
\]
\[
D = 8.8 \times 10^6, \quad 8.8 \times 10^6 = 2.6F^2
\]
\[
F^2 = \frac{3384615.385}{2.6} = 1,839.732
\]
\[
F = 1,840 \text{ gallons}
\]