COMPLETING THE SQUARE – PRACTICE QUESTIONS

metatutor

1.
The equation $x^2 + 4x + 9$ can be written in the form $(x + a)^2 + b$.
Find a and b.

$(x+2)^2 + 9 - 2^2 = (x+2)^2 + 5$

$a = 2, b = 5$

2.
Complete the square: $x^2 + 10x - 1$

$(x+5)^2 - 1 - 5^2 = (x+5)^2 - 26$

3.
Complete the square: $x^2 - 8x + 13$

$(x-4)^2 + 13 - 4^2 = (x-4)^2 - 3$

4.
Complete the square: $x^2 + 12x + 45$

$(x+6)^2 + 45 - 36 = (x+6)^2 + 9$

5.
Complete the square: $x^2 - 6x - 17$

$(x-3)^2 - 17 - 3^2 = (x-3)^2 - 26$

6.
Complete the square: $x^2 - 20x + 97$

$(x-10)^2 + 97 - 10^2 = (x-10)^2 - 3$

7.
Complete the square: $x^2 + 3x + 7$

$$\left(x+\tfrac{3}{2}\right)^2 + 7 - \left(\tfrac{3}{2}\right)^2 = \left(x+\tfrac{3}{2}\right)^2 + 7 - \tfrac{9}{4}$$
$$= \left(x+\tfrac{3}{2}\right)^2 + \tfrac{28}{4} - \tfrac{9}{4}$$
$$= \left(x+\tfrac{3}{2}\right)^2 + \tfrac{19}{4}$$

8.
Complete the square: $x^2 + 5x + 11$

$$\left(x+\tfrac{5}{2}\right)^2 + 11 - \left(\tfrac{5}{2}\right)^2 = \left(x+\tfrac{5}{2}\right)^2 + 11 - \tfrac{25}{4}$$
$$= \left(x+\tfrac{5}{2}\right)^2 + \tfrac{44}{4} - \tfrac{25}{4}$$
$$= \left(x+\tfrac{5}{2}\right)^2 + \tfrac{19}{4}$$

9.
Complete the square: $x^2 - 7x + 12$

$$\left(x-\tfrac{7}{2}\right)^2 + 12 - \left(\tfrac{7}{2}\right)^2 = \left(x-\tfrac{7}{2}\right)^2 + 12 - \tfrac{49}{4}$$
$$= \left(x-\tfrac{7}{2}\right)^2 + \tfrac{48}{4} - \tfrac{49}{4}$$
$$= \left(x-\tfrac{7}{2}\right)^2 - \tfrac{1}{4}$$

10.
Complete the square: $x^2 - 9x - 4$

$$\left(x-\tfrac{9}{2}\right)^2 - 4 - \left(\tfrac{9}{2}\right)^2 = \left(x-\tfrac{9}{2}\right)^2 - 4 - \tfrac{81}{4}$$
$$= \left(x-\tfrac{9}{2}\right)^2 - \tfrac{16}{4} - \tfrac{81}{4}$$
$$= \left(x-\tfrac{9}{2}\right)^2 - \tfrac{97}{4}$$

11.
Complete the square: $x^2 + 5x + 1$

$$\left(x+\tfrac{5}{2}\right)^2 + 1 - \left(\tfrac{5}{2}\right)^2 = \left(x+\tfrac{5}{2}\right)^2 + 1 - \tfrac{25}{4}$$
$$= \left(x+\tfrac{5}{2}\right)^2 + \tfrac{4}{4} - \tfrac{25}{4}$$
$$= \left(x+\tfrac{5}{2}\right)^2 - \tfrac{21}{4}$$

12.
Find the co-ordinates of the minimum point of the graph $y = x^2 + 10x + 29$.

$(x+5)^2 + 29 - 5^2 = (x+5)^2 + 4$

$(-5, 4)$

13.
Find the co-ordinates of the minimum point of the graph $y = x^2 - 4x - 53$.

$(x-2)^2 - 53 - 2^2 = (x-2)^2 - 57$

$(2, -57)$

14.
Find the co-ordinates of the minimum point of the graph $y = x^2 + 5x - 5$.

$(x+\frac{5}{2})^2 - 5 - (\frac{5}{2})^2 = (x+\frac{5}{2})^2 - 5 - \frac{25}{4}$

$= (x+\frac{5}{2})^2 - \frac{20}{4} - \frac{25}{4}$

$= (x+\frac{5}{2})^2 - \frac{45}{4}$

$(-\frac{5}{2}, -\frac{45}{4})$

15.
Find the co-ordinates of the minimum point of the graph $y = x^2 + 9x - 8$.

$(x+\frac{9}{2})^2 - 8 - (\frac{9}{2})^2 = (x+\frac{9}{2})^2 - 8 - \frac{81}{4}$

$= (x+\frac{9}{2})^2 - \frac{32}{4} - \frac{81}{4}$

$= (x+\frac{9}{2})^2 - \frac{113}{4}$

$(-\frac{9}{2}, -\frac{113}{4})$

16.
The equation $2x^2 + 8x + 7$ can be written in the form $a(x + b)^2 + c$.
Find a, b and c.

$$2(x^2+4x)+7 = 2(x+2)^2+7-2\times2^2$$
$$= 2(x+2)^2+7-8$$
$$= 2(x+2)^2-1$$

$$a=2, b=2, c=-1$$

17.
Complete the square: $2x^2 + 4x - 11$

$$2(x^2+2x)-11 = 2(x+1)^2-11-2\times1^2$$
$$= 2(x+1)^2-11-2$$
$$= 2(x+1)^2-13$$

18.
Complete the square: $3x^2 + 12x - 13$

$$3(x^2+4x)-13 = 3(x+2)^2-13-3\times2^2$$
$$= 3(x+2)^2-13-12$$
$$= 3(x+2)^2-25$$

19.
Complete the square: $2x^2 - 16x + 39$

$$2(x^2-8x)+39 = 2(x-4)^2+39-2\times4^2$$
$$= 2(x-4)^2+39-32$$
$$= 2(x-4)^2+7$$

20.

Complete the square: $2x^2 + 6x + 11$

$$2(x^2+3x)+11 = 2(x+\tfrac{3}{2})^2+11-2\times(\tfrac{3}{2})^2$$
$$= 2(x+\tfrac{3}{2})^2+11-2\times\tfrac{9}{4}$$
$$= 2(x+\tfrac{3}{2})^2+11-\tfrac{9}{2}$$
$$= 2(x+\tfrac{3}{2})^2+\tfrac{22}{2}-\tfrac{9}{2}$$
$$= 2(x+\tfrac{3}{2})^2+\tfrac{13}{2}$$

21.

By completing the square, show that a solution to the equation $x^2 + 8x + 1 = 0$ is $\sqrt{15} - 4$.

$$(x+4)^2+1-4^2=0$$
$$(x+4)^2-15=0$$
$$(x+4)^2=15$$
$$x+4=\sqrt{15}$$
$$x=\sqrt{15}-4$$

22.

By completing the square, show that a solution to the equation $x^2 + 7x + 9 = 0$ is $\frac{\sqrt{13}-7}{2}$.

$$(x+\tfrac{7}{2})^2+9-(\tfrac{7}{2})^2=0$$
$$(x+\tfrac{7}{2})^2+9-\tfrac{49}{4}=0$$
$$(x+\tfrac{7}{2})^2+\tfrac{36}{4}-\tfrac{49}{4}=0$$
$$(x+\tfrac{7}{2})^2-\tfrac{13}{4}=0$$
$$(x+\tfrac{7}{2})^2=\tfrac{13}{4}$$
$$x+\tfrac{7}{2}=\sqrt{\tfrac{13}{4}}$$
$$x=\tfrac{\sqrt{13}}{2}-\tfrac{7}{2}=\tfrac{\sqrt{13}-7}{2}$$

23.

By completing the square, show that a solution to the equation $2x^2 - 6x + \frac{5}{2} = 0$ is $\frac{5}{2}$.

$$2(x^2-3x)+\frac{5}{2}=0$$
$$2(x-\tfrac{3}{2})^2+\tfrac{5}{2}-2\times(\tfrac{3}{2})^2=0$$
$$2(x-\tfrac{3}{2})^2+\tfrac{5}{2}-\tfrac{9}{2}=0$$
$$2(x-\tfrac{3}{2})^2-2=0$$
$$2(x-\tfrac{3}{2})^2=2$$
$$(x-\tfrac{3}{2})^2=1$$
$$x-\tfrac{3}{2}=1$$
$$x=1+\tfrac{3}{2}=\frac{5}{2}$$

24.

(a) Find the co-ordinates of the minimum point of the graph $y - 2 = 3x^2 + 24x + 5$.

$$y=3x^2+24x+7$$
$$=3(x^2+8x)+7$$
$$=3(x+4)^2+7-3\times4^2$$
$$=3(x+4)^2+7-48$$
$$=3(x+4)^2-41$$

$$(-4,-41)$$

(b) Hence find the equation of the line of symmetry of the graph $y - 2 = 3x^2 + 24x + 5$.

$$x=-4$$